

DISK-STABILITY CONSTRAINTS ON THE NUMBER OF ARMS IN SPIRAL GALAXIES

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ABSTRACT

A model based on disk-stability criteria to determine the number of spiral arms of a general disk galaxy with an exponential disk, a bulge and a dark halo described by a Hernquist model is presented. The multifold rotational symmetry of the spiral structure can be evaluated analytically once the structural properties of a galaxy, such as the circular speed curve, and the disk surface brightness, are known. By changing the disk mass, these models are aimed at varying the critical length scale parameter of the disk and lead to a different spiral morphology in agreement with prior models. Previous studies based on the swing amplification and disk stability have been applied to constrain the mass-to-light ratio in disk galaxies. This formalism provides an analytic expression to estimate the number of arms expected by swing amplification making its application straight-forward to large surveys. It can be applied to predict the number of arms in the Milky Way as a function of radius and to constrain the mass-to-light ratio in disk galaxies for which photometric and kinematic measurements are available, like in the DiskMass survey. Hence, the halo contribution to the total mass in the inner parts of disk galaxies can be inferred in light of the ongoing and forthcoming surveys.

Subject headings: Galaxy: disk - Galaxy: evolution - galaxies: kinematics and dynamics - stars: kinematics and dynamics

1. INTRODUCTION

A long-standing problem in understanding the dynamics of disk galaxies is the uncertainty of the fractional contribution of the dark halo to the total mass of the galaxy within the optical radius of the stellar disk. While galaxies are believed to be dark matter dominated in their outer parts, at radii much larger than the optical radius, the assessment of the relative fraction of baryons and dark matter in the inner part of disk galaxies is still debated (e.g. Courteau et al. 2014).

The circular-speed curve constrains the total mass distribution at all radii, and the disk contribution is usually estimated by combining the light profile with an estimate of the mass-to-light ratio of the disk. The difference between the total rotation curve and the circular velocity of the disk is assumed to be due to the presence of dark matter (Bosma 1978; Carignan & Freeman 1985). Because the mass-to-light ratio of the disk is uncertain, the relative contributions of the disk and halo are poorly determined.

Attempts to constrain the mass-to-light ratio come from studies of stellar populations (e.g. Conroy 2013), and comparisons between dynamical and stellar population mass estimates (Bell & de Jong 2001). However, these models depend on assumptions made about the IMF and stellar ages, leading to rather large uncertainties in the disk contribution. A maximum disk hypothesis for the mass decomposition has been suggested for disk galaxies (van Albada et al. 1985). It assumes that the total rotation curve of disk galaxies out to the optical radius can be fitted by models that assume a mass-to-light ratio of the disk independent of radius and with no need for dark matter in the inner parts (maximum disk). A typical maximum disk contributes more than

80% of the total circular speed at $2.2 R_d$, where R_d is the scale length of the exponential disk. In terms of disk mass fraction the maximum disk hypothesis requires a disk contribution of $f_d(2.2 R_d) = (V_{disk}/V_{tot})^2 > 0.7$.

Additional arguments have been made to constrain the halo and disk contribution to the total gravitational field via different techniques. In particular, studies on Tully-Fisher residuals applied to bright disk galaxies suggest that on average disk galaxies are sub-maximal with $f_d(2.2 R_d) = 0.25-0.5$ (Courteau & Rix 1999). Another approach assumes that the peak circular velocity of the stellar disk, measured at $R = 2.2 R_d$, is related to the vertical velocity dispersion, and the scale height through some scaling relations (van der Kruit 1988). More recently, Bershadsky et al. (2010) applied the velocity dispersion technique to a sample of 46 nearly face-on galaxies finding that the disk fraction of their sample ranges between 0.16 and 0.5. A similar method has been recently applied to the Milky Way. The analysis includes the ability to disentangle stellar populations and results in a low estimate of the disk scale length and a correspondingly high disk fraction of 0.69 ± 0.07 (Bovy & Rix 2013) (hereafter BR13).

The study here presents a model based on swing amplification (Toomre 1981) to compute analytically the azimuthal wavenumber at each radius that is likely to experience the strongest growth to determine the number of spiral arms expected at that radius (Athanassoula 1988; Toomre & Kalnajs 1991; Bosma 1999; Fuchs & Athanassoula 2005). For a sample of grand design spirals, Athanassoula et al. (1987), hereafter ABP87, applied the swing amplification formalism in an attempt to constrain the mass-to-light ratio of dozens of disks. ABP87 study is of considerable observational interest, since it treated the disk stars and gas as distinct components, and computed their contribution of the total rotation curve using directly their observed projected sur-

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face density profiles with an appropriate potential solver. The bulge contribution was estimated in a similar way. This approach offered the advantage not to rely on a specific disk, bulge or halo model. It constrained the mass-to-light ratio of disk galaxies and thereby inferred the dark matter and disk contribution within the optical radius. Here the disk-stability criterion is generalized for a galaxy characterized by an exponential disk, a stellar bulge, and a Hernquist halo. It provides a simple analytic expression to estimate the number of arms expected by swing amplification, making its application straightforward to large surveys.

The model is detailed in Section 2. Numerical simulations are presented in Section 3. Section 4 applies the method to an observational sample, and Section 5 summarizes the main results.

2. MODEL

We consider the case of a stable disk with $Q \geq 1$ responding to gravitational perturbations by swing amplification. For a given disk the efficiency of the amplification is characterized by a factor defined as:

$$X = \frac{\lambda}{\lambda_{crit}} = \frac{\kappa^2}{2\pi G \Sigma} \frac{R}{m} \quad (1)$$

where the critical wavelength is defined as $\lambda_{crit} = 4\pi^2 G \Sigma / \kappa^2$, G is the gravitational constant, and κ is the epicycle frequency.

Numerical studies have shown that for a stable disk, as arms swing from leading to trailing, the amplification factor depends on the value of the X parameter. Particularly suggestive is Fig. 8 of Toomre (1981) showing that for disks with flat circular velocity curves and $Q = 1.2$ the gravitational response to perturbations of a stellar disk is the same as that of a gaseous disk and that the swing amplification is strongest when the parameter X is 1.5, leading to an amplification of a factor from 40 up to 100 (Athanasoula 1984; Fuchs 2001).

Given the circular speed curve, the epicyclic frequency of the galaxy disk is determined. For a general galaxy with a bulge, disk and a dark halo, the epicyclic frequency depends on the total angular frequency, which is the sum of the angular frequencies, according to:

$$\Omega^2 = \Omega_D^2 + \Omega_B^2 + \Omega_H^2 \quad (2)$$

For an exponential disk, the angular frequency is:

$$\Omega_D^2 = \frac{GM_D}{2R_h^3} [I_0(y)K_0(y) - I_1(y)K_1(y)] \quad (3)$$

where $y = R/2R_d$, R_d is the disk scale length, and M_D is the disk total mass, including gas and stellar components. For bulges and halos described by Hernquist models (Hernquist 1990), the angular frequency of the bulge is:

$$\Omega_B^2 = \frac{GM_B}{R(R + a_b)^2} \quad (4)$$

where a_b and M_B are the bulge scale length and mass, respectively, and for the halo:

$$\Omega_H^2 = \frac{GM_H}{R(R + a_h)^2} \quad (5)$$

where a_h and M_H are the halo scale length and the mass, respectively.

The Fourier coefficient, m , that characterizes the number of spiral arms, generalized for a galaxy with those properties becomes:

$$m = \frac{e^{2y}}{X} \left(\left[\frac{M_B}{M_D} \frac{2y + 3a_b/R_d}{(2y + a_b/R_d)^3} \right] + \left[\frac{M_H}{M_D} \frac{2y + 3a_h/R_d}{(2y + a_h/R_d)^3} \right] + \frac{y^2}{2} (3I_1K_0 - 3I_0K_1 + I_1K_2 - I_2K_1) + 4y(I_0K_0 - I_1K_1) \right) \quad (6)$$

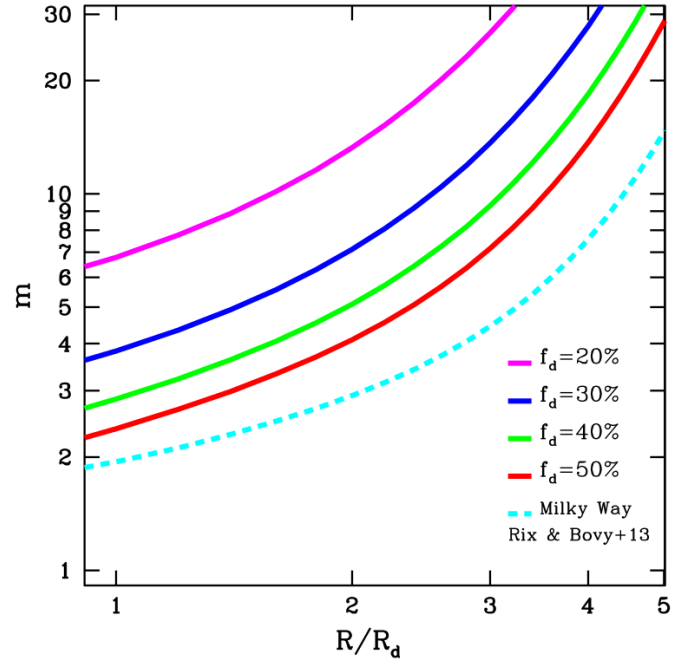


FIG. 1.— The total number of arms as a function of the distance from the center for a galaxy with the halo mass of $9.5 \times 10^{11} M_\odot$ and a halo scale length of 29.6 kpc. Galaxies with the same mass for the halo display a progressive increase of the disk fraction, f_d , from 20% to 50%, from the top to bottom. The predicted number of arms for the Milky Way is also displayed adopting the structural properties of Bovy & Rix (2013) with the inclusion of a bulge with a mass of $4 \times 10^9 M_\odot$ and a scale radius of 600 pc. The assumed amplification factor is $X=1.5$.

For a galaxy with a halo mass of $9.5 \times 10^{11} M_\odot$ the total number of arms m as a function of the distance from the galaxy center is displayed in Fig. 1. The different lines display models where the total mass distribution of the dark halo is kept fixed but the self-gravity of the disk is progressively increased. In particular, the disk mass fraction, f_d , within 2.2 scale lengths, rises from 20% to 50%, from the top to bottom. The disk scale lengths assumed are the same as in the numerical simulations presented in the next section. No bulge is assumed for simplicity except for the Milky Way model where parameters are assumed from BR13.

Note that the total number of arms is expected to increase with distance from the galaxy center as the highest Fourier components m , which are always present, dominate the spiral structures in the outer parts of the disk. This is in agreement with previous models and comparison with observations of ABP87. However, exponential disks are less self-gravitating as we move further out from the center, thus the arms in the outer parts of the disk will have lower strength and may just be in the form of poorly amplified wakelets (D’Onghia et al. 2013).

Fig. 1 shows that at a given radius, the total number of spiral arms decreases as the disk fraction increases, as expected. The predicted number of arms of the Milky Way is also included in Fig. 1 (dashed line) adopting the structural parameters presented in BR13, with the disk fraction being estimated to be $f_d \approx 0.69$ within $2.2 R_d$. Note that according to this analysis the Milky Way should have two spiral arms at approximately 4.50 kpc, nearly at the edge of the stellar bar, and a total of 5-6 spiral arms, lower in strength, in the solar neighborhood.

3. NUMERICAL EXPERIMENTS

Numerical simulations of various disk models have shown that changes to the disk mass lead to varying the critical length scale parameter λ_{crit} and lead to a different spiral morphology (see e.g. Sellwood & Carlberg (1984); Carlberg & Freedman (1985); Nelson et al. (2012)).

Here a set of numerical simulations are employed to determine the extent to which the response of the disk depends specifically on the mass distribution of the galaxy and on the disk self-gravity.

In order to test the formalism introduced above, a set of numerical simulations are carried out with the parallel TreePM code GADGET-3 (last described in Springel (2005)). Following D’Onghia et al. (2013), the tree-based gravity solver is employed coupled with a static potential to solve for the evolution of collisionless particles. The galaxies in this study consist of dark matter halos and rotationally supported stellar disks of 10^7 particles. The parameters describing each component are independent and the models are constructed similarly to the approach described in Hernquist (1993). The dark matter mass distribution is modeled assuming a Hernquist profile with a total halo mass and a scale length of $9.5 \times 10^{11} M_\odot$ and 29.6 kpc, respectively. The stellar disk is modeled according to the initial conditions as a thin exponential surface density profile of scale length R_d , so that the stellar disk mass is $M_* = f_d M_{tot}$ where f_d is the disk fraction of the total mass of the galaxy. The vertical mass distribution of the stars in the disk is specified by the profile of an isothermal sheet with a radially constant scale height z_0 . A value of $z_0 = 0.1 R_d$ is adopted.

In a sequence of experiments shown in Fig. 2, the contribution of the disk to the total mass of the galaxy is altered. Like in Fig. 1 the total mass of the halo is fixed but the disk mass is increased in order to obtain a disk fraction, f_d , within 2.2 disk scale lengths varying from 20% to 50% of the total mass. Disk scale lengths are set in the initial conditions to values that conserve the total angular momentum of the disk. They range from 2.75 kpc in the model with $f_d=0.5$ to 3.3 kpc in the one with $f_d=0.2$. Fig. 2 shows that the galaxy ranges from the multi-armed spirals obtained for disks with low mass fractions (lower panels in Fig. 2), to those with

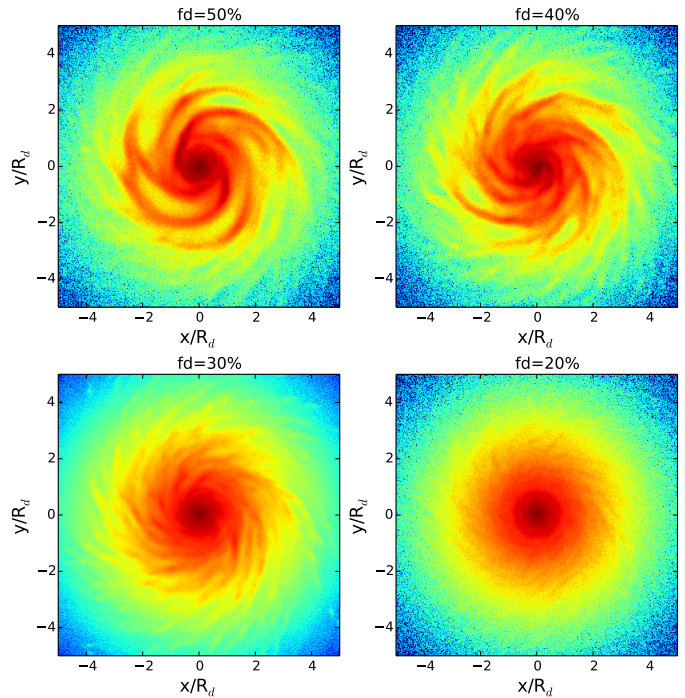


FIG. 2.— Spiral morphologies of the simulated stellar disk displayed face-on after one galactic year. The disk mass fraction at 2.2 scale lengths, f_d ranges from 50% of the total mass (top left panel) to 20% (bottom right panel).

four prominent arms typical of galaxies where the disk contributes about 50% of the total mass (top left panel in Fig. 2). As anticipated by the analytic arguments and previous studies, the results shown in Fig. 2 confirm that the spiral morphology in the simulated disks is indeed determined by the structural properties of the galaxy through the expression of λ_{crit} .

A quantitative analysis of the strength of the arms as a function of the galaxy radius has been performed through a Fourier analysis of the surface mass density of the stellar disk. The results show that the amplitude of the arms, on average, ranges from 10% of the stellar background for the multi-armed galaxies (right bottom panel of Fig. 2) to 50% for the galaxies with four arms and approximately four-fold rotational symmetry (top left panel in Fig. 2). The number of arms increases as a function of radius in agreement with the analytic estimates.

4. APPLICATION TO THE DISKMASS SAMPLE

The formalism introduced can then be applied to the Milky Way and to the DiskMass sample, consisting of face-on galaxies with rotation velocities between 100 kms^{-1} and 250 kms^{-1} (Bershady et al. 2010; Davis et al. 2015). Models are run assuming the structural properties of the galaxies, when available, in particular scale lengths, and total mass as reported by Martinsson et al. (2013), with the observed disk fraction f_d ranging from 0.16 to 0.5. As expected, Fig. 3 shows a correlation between the total number of spiral arms predicted by the swing amplification and the disk fraction. Submaximal disks are thus expected to be multi-armed galaxies, and the two-armed galaxies are the ones with higher f_d within 2.2 scale lengths. A visual inspection of the images of the

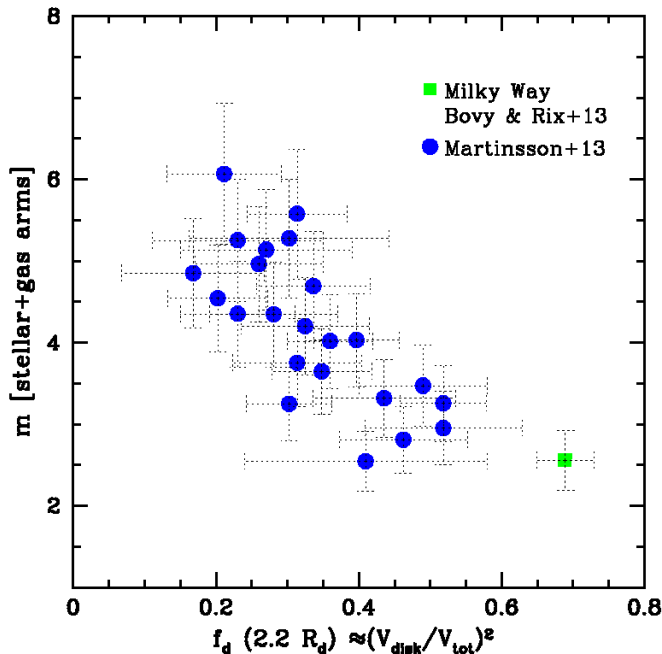


FIG. 3.— Total number of spiral arms predicted by models applied to the DiskMass catalog of galaxies (blue symbols) as a function of the disk fraction, f_d , measured at 2.2 disk scale lengths. m is the average value estimated by models assuming $1.5 \leq X \leq 2$ with the error bars given by the two extreme values. The prediction of the number of spiral arms of the Milky Way is also included (green symbol) for the structural parameters of the Milky Way reported in Bovy & Rix (2013).

galaxy sample in the available bands supports this result, although a more quantitative analysis is needed.

5. CONCLUSIONS

A model based on the swing amplification mechanism is generalized for a realistic galaxy model with an exponential stellar disk and a dark halo described by the Hernquist mass profile. The number of spiral arms is derived in the context of disk stability. More generally, the model confirms previous results that show that the number of arms in galaxies should depend only on the

structural properties of the galaxy, with a correlation between the disk fraction and the dominant wave mode numbers. These models are aimed at providing an additional constraint on the number of spiral arms in the Milky Way as a function of the radius and an independent test on the structural parameters of the Milky Way provided by BR13 (as shown in Fig.3). They have been applied to the DiskMass survey to confirm the known correlation between the total number of spiral arms predicted by the swing amplification theory and the disk mass fraction.

Ongoing and forthcoming surveys of disk galaxies provide strong tests of the applicability of these models. In particular, they can constrain the mass-to-light ratio of disk galaxies and hence to infer the dark matter and disk contribution within the optical radius. Although the study presented in ABP87 is model independent and with a more accurate treatment of the swing amplification, the formalism presented here has the advantage to provide an analytic expression for the number of arms expected by swing amplification that can be used to compare with the actual number of arms observed in large sample of galaxies. For a given rotation curve the epicycle frequency is determined analytically. If the number of arms expressed by m at a given radius can be measured independently, that number indicates the azimuthal wavenumber undergoing the strongest swing amplification at that radius (according to eq.(6) for $X = 1.5 - 2$ for any given galaxy). Therefore the surface density of the disk can be derived analytically from eq.(1), leading to an independent constraint in the mass-to-light ratio of the disk, and hence to the halo contribution to the total mass.

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